Estimating Traffic Flow Rate on Freeways from Probe Vehicle Data and Fundamental Diagram

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***Abstract*—Traffic flow rate information obtained from loop detectors is important for many traffic management applications. Given the installation and maintenance costs of such detectors is high, many transportation agencies are shifting to probe vehicle based traffic data. However, estimating traffic flow rates from probe vehicle data remains critical. This paper attempts to estimate traffic flow rates by utilizing a well-calibrated fundamental diagram in combination with the traffic speed information obtained from the probe vehicle. Different single-regime fundamental diagrams and aggregation intervals of the probe vehicle data are investigated in search of the combination that provides the most accurate estimate of flow rates. The results suggest that flow rates are best estimated by using fundamental diagram developed by Van Aerde. Moreover, estimates of flow rates during congested periods are found to be more accurate than free-flow periods.**

Keywords—probe vehicle, fundamental diagram, flow estimation

I. Introduction

Accurate and reliable estimation of traffic flow rates are important for a number of applications in traffic management and control, e.g., assessing the freeway quality of service, examining the vehicle arrival rate for evaluating shockwaves due to drop in capacity or incidents, optimization of signals at intersections etc. Traditionally, traffic information collected from loop detectors or radar sensors at a fixed point are used for measuring traffic flow rates. Given the installation and maintenance costs of such devices are high, many transportation agencies are focusing on obtaining traffic information from probe vehicle (PV) data generated from cellphones or navigation devices [1]. Another advantage of using PV data is the superiority in coverage as traffic information along the entire road network can be collected.

The data collected from PV consist of the location of the PV and their spot speeds taken at short time intervals, e.g. every second or fraction of a second, which enables reconstructing the trajectory of the vehicles. Such data have been used for a variety of purposes, e.g., queue length estimation at signalized intersections [2, 3], analysis of travel time [4] and traffic state estimation [5]. Determining traffic flow rates from the PV data on a real time basis is a challenging process as the penetration rate of PVs can be relatively low. However, this challenge can be tackled by utilizing a fundamental diagram (FD) in combination with the traffic speed information obtained from PV data. Besides the recent work of [6], estimation of traffic flow rates from PV data has not received much attention.

A FD provides a relationship among the macroscopic traffic parameters, namely: speed (u), volume (q) and density (k). Given a robust and well-calibrated FD corresponding to the road segment of interest, one can estimate the volume of traffic corresponding to the speed obtained from PV. This approach of estimating traffic flow rates heavily depends on the goodness of fit of FD to the traffic data and the aggregation interval of the PV data. This paper focuses on comparing the estimates of traffic flow rates obtained from a variety of singe-regime FDs and aggregation intervals of the PV data. The FDs considered in this paper are: 1) Greenshield, 2) Underwood, 3) Northwestern. and 4) Van Aerde. The aggregation intervals of the PV data considered are: 1) 5 minutes, 2) 10 minutes, and 3) 15 minutes. Other FDs and aggregation intervals are beyond the scope of this paper. The accuracy of the estimated traffic flow rate from the different combinations of FDs and aggregation intervals of the PV data are reported in terms of relative percentage errors.

The remainder of the paper is organized as follows. Following this introductory section, review of literature is presented. This is followed by discussion on the methodological approach and the data used for the analyses. Finally, the results are discussed, conclusions are drawn, and insights on future works are presented.

II. Literature Review

The earliest concept of probe vehicle (PV) was perhaps introduced by Wardrop and Charlesworth [7]. They recorded the number of vehicles being passed by the moving vehicle and passing the moving vehicle. They also recorded the number vehicles in the other direction and the travel time of the moving vehicle. All of which were used to estimate flow and speed.

Several decades later, as PV technology becomes available, a handful of studies have been performed in estimating flow from PV data. Neumann et al [6] applied the Van Aerde [8] fundamental diagram (FD) to estimate hourly traffic flow using the speed of PV as an input. They then extended the study by adding Bayesian statistics to the analyses [9]. Further research looked into flow estimation by considering the spacing between the PV and the vehicle in front of it [5].

PV based studies have also been used for various analyses of traffic conditions such as estimating queue length and travel time. The trajectories of the PV combined with the shockwave theory [10, 11] are used to estimate queue lengths [12-15]. While other research have looked into segment travel time and/or speed for a case study in Israel [16] and by applying Markov chain technique to NGSIM data [17]. Instead of focusing on a single attribute, Hiribarren and Herrera [18] and Nantawichit et al. [4] study estimation of several traffic attributes such as travel time, speed and/or queue lengths combined.

Using taxis as PVs, Donovan and Work [19] studied the resiliency of New York city transportation network by considering the PVs travel time before and after hurricane Sandy. In an experiment called Mobile Century, a team from UC Berkeley developed a traffic monitoring system based on PV data [20].

Transportation agencies are looking at alternative sources of traffic data. The Ohio Department of Transportation switched to third party traffic data instead of loop detectors for traffic management purposes [1]. Third parties are private companies that collect PV data and selling them to the public and private agencies. The data may come from a variety of sources such as fleet vehicles. In the future it is expected that more transportation agencies will shift from loop detectors to PVs as a source of traffic data.

This paper aims at estimating volume of traffic from FD by using speed obtained from PV. Similar concept has previously been applied by [6]. However, their work had critical limitation as they used a typical FD for roads of the same type, e.g., same FD for all freeways. This fails to capture the spatial variations of traffic, e.g., difference in proportion of HGVs, presence or absence of on-off ramps and bottlenecks that restrict traffic flow. Moreover, they considered fitting only one type of FD which is the Van Aerde FD. To complement this limitation, four different single-regime FDs were applied by fitting them to traffic data corresponding to the same corridor where the PV data was collected. In addition, the PV data used in this paper was aggregated at a 5 minute interval which provides better temporal resolution.

III. METHODOLOGY

A FD describes the relationship between traffic speed (*u*), flow (*q*) and density (*k*). The methodology propose in this paper utilize the speed-flow relationship, i.e., flow (*q*) is estimated when speed (*u*) is known. Four different single-regime FDs are considered, namely Greenshield [21], Underwood [22], Northwestern [23] and VanAerde [8], and the estimated flow (*q*) for a given speed (*u*) obtained from each of the FDs are compared. Only single-regime FDs are used in this paper so that the problem of continuity at the boundary of the regimes of multi-regime FDs is avoided.

The speed-density relationship of the four FDs and their transform functions are listed in Table I. From the speed-density relationships and the regression analysis of the transform functions other relationships such as the speed-flow and flow-density can be estimated.

Table I. FUNDAMENTAL DIAGRAM RELATIONSHIP

Model	Speed-Density Relationship	Regression
Green-shield	$k = k_j\left(1 - \frac{u}{u_f}\right)$	$k = k_j - \frac{k_j}{u_f}u$
Under-wood	$k = k_o \ln\left(\frac{u_f}{u}\right)$	$k = k_o \ln u_f - k_o \ln u$
North-western	$k = k_o\left(2\ln\frac{u_f}{u}\right)^{1/2}$	$k^2 = 2k_o^2 \ln u_f - 2k_o^2 \ln u$
Van Aerde	$k = \frac{1}{c_1 + \frac{c_2}{u_f - u} + c_3 u}$	$n = \frac{2u_c - u_f}{(u_f - u_c)^2}$, $c_2 = \frac{1}{k_j\left(n + 1/u_f\right)}$, $c_1 = nc_2$, $c_3 = \frac{-c_1 + \frac{u_c}{q_c} - \frac{c_2}{u_f - u_c}}{u_c}$

In Table I u_f is free-flow speed, u_c is speed at capacity, k_o is optimum density and k_j is jam density. To calculate the flow rate (*q*) in terms of speed, replace *k* which is on the left hand side of the speed-density relationships with the ratio of *q* divided by *u*. Simple algebraic manipulations transform the equations in Table I to *q*-*u* relationships.

The typical PV data consists of speed and location of the vehicle itself. Given the speed (*u*) of PV, this method would then refer to the reference speed-flow relationship of the FDs to estimate the flow rate (*q*).

In this paper, the estimates of the traffic flow rates from the different FDs are compared with the traffic flow rate measurements obtained from loop detectors which are considered to be the ground truth measurement. Percent error (PE), mean absolute percentage error (MAPE) and root mean square error (RMSE) are used as performance indicators for the deviation of the estimated flow rate from the ground truth measurements as formulated (1)-(3).

$$PE_i = \frac{F_i - O_i}{O_i} \times 100\% \tag{1}$$

$$MAPE = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{F_i - O_i}{O_i}\right| \times 100\% \tag{2}$$

$$RMSE = \frac{1}{n}\sqrt{n\sum_{i=1}^{n}(F_i - O_i)^2} \tag{3}$$

where: F_i is the i^{th} estimate value
O_i is the i^{th} observe value
n is the number of samples

IV. CASE STUDY

The study site is a corridor along I-880 northbound located near Union City between Stevenson Blvd. and Winton Ave. ramps in the San Francisco bay area. PV data were collected from 10:00am to 6:00pm on Friday February 8th 2008. This data collection effort was a part of the Mobile Century project [20]. In this experiment, 165 drivers were recruited to drive PV along the study site. The PV were equipped with GPS-enabled cell phones which on average transmitted data every 3 seconds. Data collected were latitude, longitude and timestamp. Post processing by the Mobile Century team added speed and postmile to the PV data. The penetration rate of PV is in the range of 2-5% depending on time of the day.

Even though the original Mobile Century project covered a considerably large corridor, the penetration rate of PV was low in the afternoon hours. In addition, a crash was reported during mid-morning between postmile 26 and 27. Therefore, a smaller segment around postmile 25 which experienced a recurrent congestion was selected for this study. Fig. 1 illustrates the study area's PV trajectories, several loop detector locations which are identified by the horizontal dotted lines and the resulting shockwave speed *w* from the mid-morning crash indicated by dotted red line. *w* is the slope of the linear regression line for PV that encountered speed decrease.

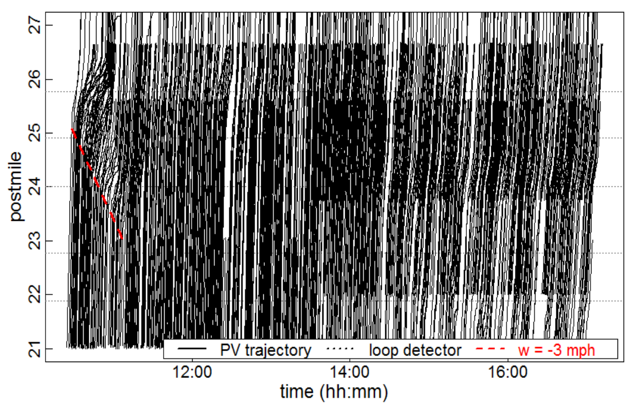

Fig. 1. Probe vehicle trajectories

Loop detector data, corresponding to the segment postmile 25 where PV data were collected, were retrieved from California Department of Transportation website [24]. The downloaded data included flow and speed for each lane aggregated every 5-min for thirty four days starting from Jan 6th to Feb 9th 2008. These data were aggregated across all lanes and were used to develop the four FDs as shown in Fig. 2. In this figure, each line represented each FDs with the observed data in the background. Of the three relationships, the most relevant to this research is Fig. 2b which is the speed-flow relationship.

To better understand the proposed methodology, a thirty minute time period analyses of the loop detector and PV data are shown in Fig. 3. In Fig. 3a, the location of the loop detector nearest to postmile 25 is identified as the horizontal dotted line. Each PV data entries are identified as circles with linear lines connecting them. Each color and line indicate a unique PV.

Speed data from loop detector and PV are displayed on Fig. 3b. In this figure, the speeds for each PV data entry are shown as circles. Again, each color indicates a unique PV. The average speeds from PV aggregated every 5-min are shown as diamond shapes while the average speeds from loop detector are shown as squares with intersecting diagonals. These average speeds from PVs are entered into the FDs to estimate the corresponding flow rates.

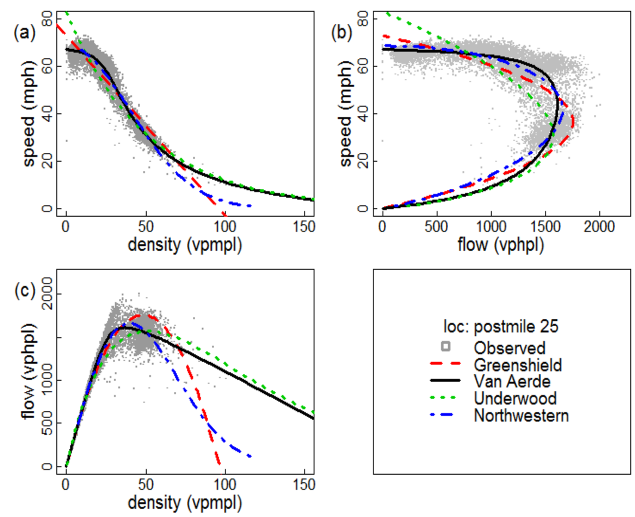

Fig. 2. Different models of fundamental diagrams

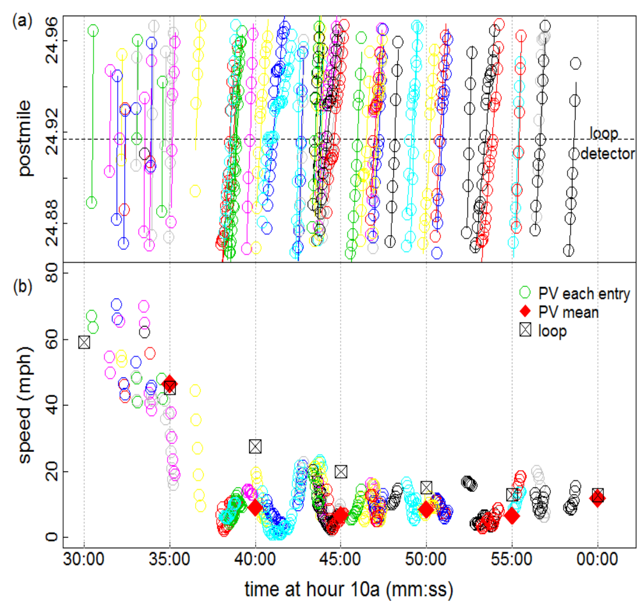

Fig. 3. Speed data from probe vehicles (5-min aggregation)

V. RESULTS

As discussed previously, the estimation of traffic flow rate is conducted by utilizing different FDs and traffic speed obtained from PV data. Fig. 4 shows traffic speed information obtained from the PV data aggregated every 5-, 10- and 15-minutes. The deviations of speed of the PV aggregated in 5-, 10- and 15-minutes in terms of average PE were found to be

10%, 9%, and 13%, respectively. In general, the pattern of the speed profiles of the loop detectors is well maintained by the speed obtained from the PV data.

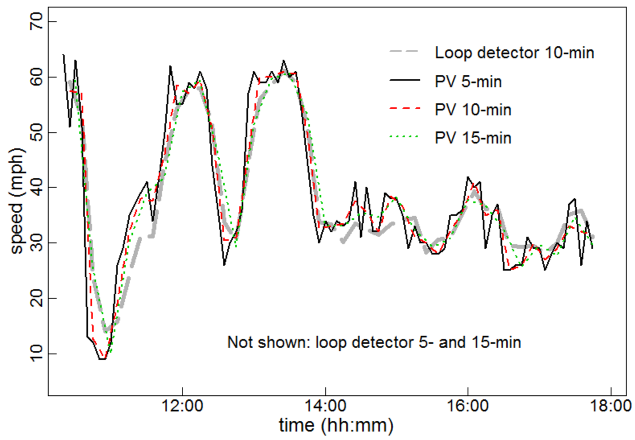

Fig. 4. Probe vehicle and loop detector speed comparison

Fig. 5 shows the estimates of traffic flow rates from different FDs and aggregation intervals of the PV speeds by time of the day. It is noticed that the deviations of the estimated flow rates from the observed values are lower when volume is at capacity or close to capacity while it is higher when traffic is mild. Moreover, it is observed that the Greenshield, Underwood and Northwestern models tend to underestimate flow rates during medium flow rates. On the other hand, Greenshield model tends to overestimate the flow rate when flow rates are near capacity. Overall, Van Aerde model provides reasonable estimate of traffic flow rates with good estimates during periods of high flow rates with slight under estimation during medium flow rates.

The baseline calculation of 5-min is then aggregated into 10- and 15-min flow rate. The difference in flow for the time aggregations are shown in Fig. 5a (5-min), b (10-min) and c (15-min). Overall it is observed that there is less fluctuation as the time aggregation increases from 5 to 15 minutes. The increase in time aggregation acted as a smoothing factor that reduced variability.

Estimation errors for each set of time aggregation and FD are examined by using different performance indicators (1)-(3), i.e., PE, MAPE and RMSE. Fig. 6 shows a summary of the magnitude of the deviations of the estimated flow rates from the observed flow rates in terms of PE for each FD and aggregation interval. The figure reveals that the magnitudes of the errors from Greenshield, Underwood and Northwestern models are high and contain significant amount of outliers. Moreover, the errors from Greenshield and Underwood models show no significant improvements with increase in aggregation interval of the PV data while Northwestern show significant reduction in estimation errors with an increase in aggregation intervals. On the other hand, the size of the boxes and whiskers of PE for Van Aerde model are smaller than the other models which suggest that the variability in the errors of estimates of the traffic flow rates is smaller. Table II lists MAPE, RMSE, average PE and standard deviation of PE for each FD model and aggregation interval. Overall, VanAerde model of FD performs the best in terms of having the smallest magnitude and variation of errors in terms of MAPE, RMSE, and average and standard deviation of PE.

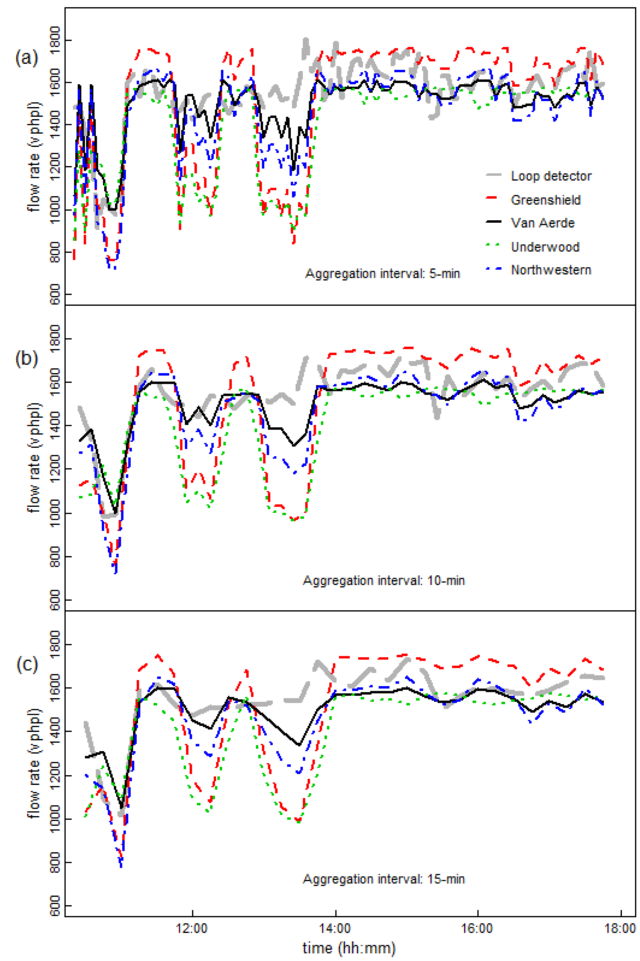

Fig. 5. Flow rate estimation from different FDs and aggregation intervals

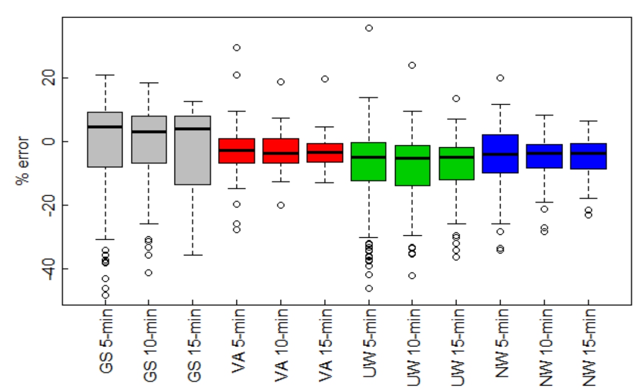

Fig. 6. Distribution of percentage error for different FDs and aggregation intervals

Table II SUMMARY OF ESTIMATION ERRORS FOR DIFFERENT FDs AND AGGREGATION INTERVALS

FD models	Aggregation interval	MAPE (abs %)	RMSE (vphpl)	Avg. Error	Std. Dev.
Greenshield	5-min	12.5	189	-2.1	17.1
	10-min	11.1	169	-2.2	15.2
	15-min	11.1	168	-2.2	14.7
Underwood	5-min	11.7	178	-8.9	14.6
	10-min	11.3	174	-9.0	13.5
	15-min	10.9	167	-9.0	12.9
Northwestern	5-min	8.7	130	-5.4	10.4
	10-min	7.1	107	-5.5	8.2
	15-min	6.8	103	-5.5	7.7
Van Aerde	5-min	6.4	98	-2.9	8.1
	10-min	5.3	83	-3.0	6.2
	15-min	5.2	79	-3.0	6.2

The distribution of the errors corresponding to the speed of the PV aggregated at 5-, 10- and 15-minutes is shown in Fig.7 . With increase in aggregation intervals the estimation errors are observed to decreased. The accuracy of flow rates estimated from Greenshield and Northwestern models are low both during low and high speed periods with relatively better accuracy during the transition from low to high speed periods. The estimation error for Underwood model is fairly stable for most of the speed range, however, excessively large errors were observed during high speed periods. For a given PV speed, Van Aerde model provides the least error compared to the other models.

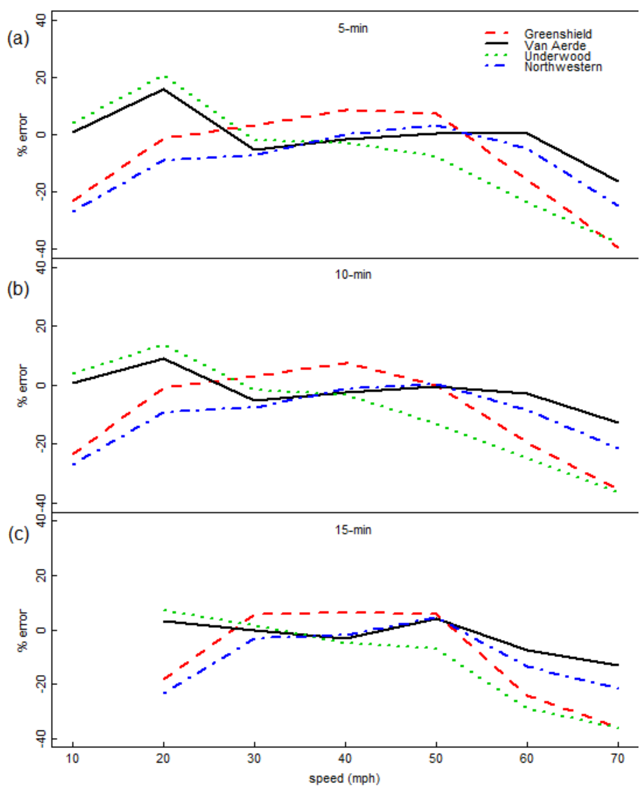

Fig. 7. Flow rate estimation error vs speed of traffic

In general, traffic flow rates are estimated more accurately during periods of low speed when compared to free-flow periods. This can be explained by the fact that traffic flow can have a wide variation during free-flow period. An extreme example would be a very low traffic flow compared to traffic flow during pre-breakdown period, all of which can occur during free-flow but vary in flow significantly. This finding was similar to a previous study [6]. During congestion, where speed was lower, the deviations are smaller than of free-flow period. This result is expected because there is less traffic flow variation when roads are congested. Vehicles tend to be closer together and have a more uniform flow making the flow rates more predictable. Considering the fact that the value or importance of traffic information during congested periods is higher than during free-flow periods, e.g., prediction of flow-break down, the proposed methodology seems to provide a promising result that can be integrated with other traffic applications for better management of traffic.

VI. CONCLUSION

Accurate estimation of traffic flow rate is critical for traffic management applications. With the wide deployment of PVs, estimating traffic flow rate from PV data is becoming vital. This study shows that the speed of PV can be used to estimate the traffic flow rate provided that a valid FD is available. Considering that there are a number of FDs to choose from, identifying the one which provides the best estimate of traffic flow rate corresponding to a given PV speed is very critical. In this regard, this paper provides the following findings:

- Out of the four single-regime FDs considered, Van Aerde model seems to provide a better result when compared to the other three models.
- With increasing aggregation interval of the PV data, better estimates of traffic flow rates can be obtained
- Traffic flow rates are more accurately estimated during congested traffic conditions compared to free-flow conditions. Considering the need for accurate estimate of traffic flow during free-flow condition is less critical, the results of the proposed methodology can be integrated with other traffic management applications.

The methodology presented in this paper heavily depends on the applied FD which itself is constructed from transformed traffic variable. Therefore, inaccuracy in the FD affects the traffic flow rate estimation. Moreover, the speed of traffic is not affected until volume reaches near capacity making the estimation of volume during free flow times challenging. The case study lacks the regime where traffic flow is very low. Considering the temporal coverage of the PV data was limited, testing the methodology on a larger PV datasets would be desirable. To overcome these limitations, data-driven approaches which take the pattern of traffic into consideration will be applied in the future to improve the estimation of flow rates from PV speed.

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